Find the 5^{th} term in the expansion of $(5n^2-7n^3)^{26}$. **SCORE: _____ / 5 PTS**

NOTES: **Your coefficient may use the operations +, –, × and positive exponents only.**
All exponents must be simplified and all signs must be explicit (at the front).

$${}_{26}C_4(5n^2)^{26-4}(-7n^3)^4 = {}_{26}C_4(5n^2)^{22}(-7n^3)^4$$

$$= \frac{26!}{4!\,22!}5^{22}n^{44}(-7)^4n^{12}$$

$$= \boxed{\frac{26\cdot25\cdot\cancel{24}\cdot23\cdot\cancel{22!}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1\cdot\cancel{22!}}}\,5^{22}7^4n^{56}$$

$$= \boxed{26\cdot25\cdot23}\,\boxed{5^{22}}\,\boxed{7^4}\,\boxed{n^{56}}$$

① POINT EACH
☆ SUBTRACT ① POINT IF YOUR ANSWER IS NEGATIVE

Use the entries of Pascal's Triangle to expand and simplify $(6z-11t)^5$. **SCORE: _____ / 5 PTS**

NOTES: **Your coefficients may use the operations +, –, × and positive exponents only.**
All exponents must be simplified and all signs must be explicit (at the front).
You must show the intermediate step in the expansion to get full credit.

$$1(6z)^5(-11t)^0 + \boxed{5(6z)^4(-11t)^1} + \boxed{10(6z)^3(-11t)^2} + \boxed{10(6z)^2(-11t)^3}$$
$$\boxed{+\,5(6z)^1(-11t)^4} + 1(6z)^0(-11t)^5$$

$$= \boxed{6^5z^5}\ \boxed{-\,5\cdot6^4\cdot11z^4t}\ \boxed{+10\cdot6^3\cdot11^2z^3t^2}\ \boxed{-\,10\cdot6^2\cdot11^3z^2t^3}$$
$$\boxed{+\,5\cdot6\cdot11^4zt^4}\ \boxed{-11^5t^5}$$

½ POINT EACH ☆ SUBTRACT ① POINT IF YOUR NEGATIVES IN THE FINAL ANSWER ARE INSIDE ( )'S.

Find the rational number representation of the repeating decimal $0.3\overline{45}$. **SCORE: _____ / 5 PTS**

NOTES: **Only the 45 is repeated.**
You must use only techniques from sections 9.2-9.5.

$$\boxed{0.3 + 0.045 + 0.00045 + 0.0000045 + \cdots}\ ①$$

$$= \frac{3}{10} + \boxed{\frac{\frac{45}{1000}}{1-\frac{1}{100}}}\ (1\tfrac{1}{2})$$

$$= \frac{3}{10} + \frac{\frac{45}{1000}}{\frac{99}{100}}$$

$$= \frac{3}{10} + \frac{\cancel{45}^{5}}{\cancel{1000}_{2}}\cdot\frac{\cancel{100}}{\cancel{99}_{11}}$$

$$(\tfrac{1}{2})\ = \boxed{\frac{3}{10}} + \boxed{\frac{1}{22}}\ ①$$

$$= \frac{33+5}{110} = \frac{38}{110} = \boxed{\frac{19}{55}}\ ①$$

AJ and BJ both have sequences where the third term is 81, and the sixth term is -24. **SCORE: _____ / 5 PTS**

[a] If AJ's sequence is geometric, find the first term of AJ's sequence.

$$\begin{aligned} a_3 &= \boxed{a_1 r^2 = 81} \\ a_6 &= \boxed{a_1 r^5 = -24} \end{aligned} \ \tfrac{1}{2} \longrightarrow \frac{a_1 r^5}{a_1 r^2} = \frac{-24}{81} \longrightarrow \boxed{r^3 = -\frac{8}{27}} \ \tfrac{1}{2} \longrightarrow \boxed{r = -\frac{2}{3}} \ \tfrac{1}{2}$$

$$a_1\left(-\tfrac{2}{3}\right)^2 = 81 \longrightarrow \tfrac{4}{9}a_1 = 81 \longrightarrow \boxed{a_1 = \frac{729}{4}} \ \tfrac{1}{2}$$

[b] If BJ's sequence is arithmetic, find the sum of the first 11 terms of BJ's sequence.

$$\begin{aligned} a_3 &= \boxed{a_1 + 2d = 81} \ \tfrac{1}{2} \\ a_6 &= \boxed{a_1 + 5d = -24} \ \tfrac{1}{2} \\ &\boxed{-3d = 105} \ \tfrac{1}{2} \\ &\boxed{d = -35} \ \tfrac{1}{2} \end{aligned}$$

$$a_1 + 2(-35) = 81$$
$$a_1 = 151$$

1

$$S_{11} = \boxed{\tfrac{11}{2}(2(151) + (11-1)(-35))} \ \tfrac{1}{2}$$
$$= \tfrac{11}{2}(302 - 350) = \tfrac{11}{\cancel{2}} \cdot \underset{-24}{\cancel{-48}} = \boxed{-264}$$

Using mathematical induction, prove that $\sum_{i=1}^{n}[(2i+1)\cdot 3^i] = n \cdot 3^{n+1}$ for all positive integers n. **SCORE: _____ / 10 PTS**

BASIS STEP: PROVE $\sum_{i=1}^{1}(2i+1)\cdot 3^i = 1\cdot 3^{1+1} = 1\cdot 3^2$

$$\sum_{i=1}^{1}(2i+1)\cdot 3^i = (2(1)+1)\cdot 3^1 = 3\cdot 3 = 9$$
$$1\cdot 3^2 = 9 \ \checkmark$$

INDUCTIVE STEP: ASSUME $\sum_{i=1}^{k}(2i+1)\cdot 3^i = k\cdot 3^{k+1}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

PROVE $\sum_{i=1}^{k+1}(2i+1)\cdot 3^i = (k+1)\cdot 3^{(k+1)+1} = (k+1)\cdot 3^{k+2}$

$$\begin{aligned} \sum_{i=1}^{k+1}(2i+1)\cdot 3^i &= \sum_{i=1}^{k}(2i+1)\cdot 3^i + (2(k+1)+1)\cdot 3^{k+1} \\ &= k\cdot 3^{k+1} + (2k+3)\cdot 3^{k+1} \\ &= (k+2k+3)\cdot 3^{k+1} \\ &= (3k+3)3^{k+1} \\ &= 3(k+1)3^{k+1} \\ &= (k+1)3^{k+2} \end{aligned}$$

GRADED BY ME

BY MI, $\sum_{i=1}^{n}(2i+1)\cdot 3^i = n\cdot 3^{n+1}$ FOR ALL POSITIVE INTEGERS n